ACE
EXAM
PAPER

Student name: ____________________

2023

YEAR 12
YEARLY
EXAMINATION

Mathematics Extension 2

General Instructions

- Working time - 180 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- In Questions 11-16, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 90 marks

- Attempt Questions 11-16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1-10

1. Which of the following is an expression for $\int \frac{x}{\sqrt[3]{x^2+1}}\,dx$?

(A) $\frac{3}{2}\sqrt[3]{(x^2+1)^2}+C$

(B) $\frac{1}{2}\sqrt[3]{(x^2+1)^2}+C$

(C) $\frac{3}{4}\sqrt[3]{(x^2+1)^2}+C$

(D) $\frac{3}{2}\sqrt[3]{(x^2+1)^2}+C$

2. What is the quadratic equation with solutions $2+3i$ and $2-3i$?

(A) $z^2-4z+13=0$

(B) $z^2-4z-13=0$

(C) $z^2+4z-5=0$

(D) $z^2+4z-13=0$

3. If two vectors are $\underset{\sim}{u}=\underset{\sim}{i}-\underset{\sim}{j}-\underset{\sim}{k}$ and $\underset{\sim}{v}=4\underset{\sim}{i}+12\underset{\sim}{j}-3\underset{\sim}{k}$, what is their scalar product ?

(A) -5

(B) 19

(C) $4\underset{\sim}{i}-12\underset{\sim}{j}+3\underset{\sim}{k}$

(D) $5\underset{\sim}{i}+11\underset{\sim}{j}-4\underset{\sim}{k}$

4. Given that $z=3\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$, what is the value of $\bar{z}^3$?

(A) $9\left(\cos\frac{\pi}{2}-i\sin\frac{\pi}{2}\right)$

(B) $9\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$

(C) $27\left(\cos\frac{\pi}{2}-i\sin\frac{\pi}{2}\right)$

(D) $27\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$

5. What is the magnitude of the vector $\underset{\sim}{i} - 3\underset{\sim}{j} + 5\underset{\sim}{k}$?

(A) $\sqrt{17}$

(B) $\sqrt{35}$

(C) 17

(D) 35

6. A particle moves in simple harmonic motion such that $v^2 + 9x^2 = k$.
What is the period of the particle's motion ?

(A) $\dfrac{2\pi}{k}$ (B) 3π

(C) $\dfrac{3k}{2\pi}$ (D) $\dfrac{2\pi}{3}$

7. The converse of $P \Rightarrow Q$ is:

(A) $Q \Rightarrow P$

(B) $Q \Leftrightarrow Q$

(C) $(\text{not } P) \Leftrightarrow (\text{not } Q)$

(D) $(\text{not } Q) \Rightarrow (\text{not } P)$

8. The velocity of a body moving in a straight line is given by $v = f(x)$ where x metres is the distance from origin and v is the velocity in metres per second. The acceleration of the body in ms^{-2} is given by:

(A) $f'(x)$

(B) $f'(v)$

(C) $xf'(x)$

(D) $f(x)f'(x)$

9. What are the values of real numbers p and q such that $(2 - i)$ is a root of the equation $z^3 + pz + q = 0$?

(A) $p = -11$ and $q = -20$

(B) $p = -11$ and $q = 20$

(C) $p = 11$ and $q = -20$

(D) $p = 11$ and $q = 20$

10. What is the value of $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sqrt{1 - \cos^2 x}}\,dx$?

(A) $-\pi$

(B) $-\dfrac{\pi}{2}$

(C) π

(D) $\dfrac{\pi}{2}$

Section II

90 marks
Attempt questions 11-16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Marks**

(a) If $z = 4 - i$, express the following in the form $a + ib$ where a and b are real.

(i) $\overline{iz}$ **1**

(ii) $\dfrac{1}{z}$ **1**

(b) The point A has position vector $\overrightarrow{OA} = 2\underset{\sim}{i} + 6\underset{\sim}{j} - 3\underset{\sim}{k}$ relative to an origin O. **2**
Find a unit vector parallel to $\overrightarrow{OA}$.

(c) Find the value of $\displaystyle\int_0^{\sqrt{5}} \frac{x}{\sqrt{x^2+4}}\,dx$. **2**

(d) Let two complex numbers be $z_1 = 2\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}$ and $z_2 = 2i$.

(i) On an Argand diagram sketch the vectors OA and OB to represent z_1 and z_2 respectively. **1**

(ii) Draw the vectors $z_1 + z_2$ and $z_1 - z_2$ on the same Argand diagram. **1**

(iii) What is the exact value of $\arg(z_1 + z_2)$? **2**

(e) Find $\displaystyle\int \frac{1}{\sqrt{12 + 4x - x^2}}\,dx$ **2**

(f) Show that if $x \neq 1$ then $1 + x + x^2 + \ldots + x^n = \dfrac{x^{n+1} - 1}{x - 1}$ for $n \geq 1$. **3**

Question 12 (15 marks) **Marks**

(a) Use integration by parts to evaluate $\int x\ e^{-x} dx$ **3**

(b) (i) What is the expansion of $(1 + ia)^4$ in ascending powers of a ? **1**

(ii) Hence find the values of a such that. $(1 + ia)^4$ is real. **2**

(c) (i) Find real numbers A, B, C and D such that **2**

$$\frac{5x^3 - 3x^2 + 2x - 1}{x^2(x^2 + 1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{x^2 + 1}$$

(i) Hence find $\int \frac{5x^3 - 3x^2 + 2x - 1}{x^2(x^2 + 1)} dx$ **2**

(d) A particle is moving in simple harmonic motion with its acceleration given by:

$$\ddot{x} = -12\sin 2t$$

Initially, the particle is at the origin and has a positive velocity of 6 m/s.

(i) Find the equation for the particle's velocity. **2**

(ii) Show that $\ddot{x} = -4x$. **3**

Question 13 (15 marks) **Marks**

(a) By completing the square, find $\int \frac{1}{9x^2 + 6x + 5} dx$ **3**

(b) Lines l_1 and l_2 are given below, relative to a fixed origin O.

l_1: $\underset{\sim}{r} = \left(11\underset{\sim}{i} + 2\underset{\sim}{j} + 17\underset{\sim}{k}\right) + \lambda\left(-2\underset{\sim}{i} + \underset{\sim}{j} - 4\underset{\sim}{k}\right)$

l_2: $\underset{\sim}{r} = \left(-5\underset{\sim}{i} + 11\underset{\sim}{j} + p\underset{\sim}{k}\right) + \mu\left(-3\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}\right)$

where λ and μ are scalar parameters.

(i) Given that line l_1 and l_2 intersect, find the value of p. **3**

(ii) Hence find the point of intersection of line l_1 and l_2. **1**

(c) Show that $z = 2i, w = \sqrt{3} + i$ and $v = -\sqrt{3} - i$ are vertices of an equilateral triangle. **2**

(d) A particle is moving in a straight line in simple harmonic motion. If the amplitude of the motion is 2 cm and the period of the motion is 4 seconds, calculate the:

(i) maximum velocity of the particle. **2**

(ii) speed of the particle when it is 1 cm from the centre of the motion. **2**

(e) A particle is projected with a speed of 10 m/s and passes through a point P whose horizontal distance from the point of projection is 5 m and whose vertical height above the point of projection is 6.25 m. What is the angle of projection ? Answer correct to the nearest minute and assume g = 10 m/s^2. **2**

Question 14 (15 marks) **Marks**

(a) Express in modulus-argument form

(i) $-1+i$ **2**

(ii) $(-1+i)^n$, where n is a positive integer. **1**

(b) Given $a+b=m$, prove that, for $a>0, b>0$ and $m>0$ **3**

$$\frac{1}{a}+\frac{1}{b}>\frac{4}{m}$$

(c) A particle is projected to just clear two poles of height h metres at distances of b and c metres from the point of projection. Let v be the velocity of the projection at angle θ to the horizontal.

(i) Show that $v^2=\dfrac{(b+c)g\sec^2\theta}{2\tan\theta}$ **3**

(ii) Hence or otherwise show that $\tan\theta=\dfrac{h(b+c)}{bc}$ **2**

(iii) What is the expression in terms of h, b and c, for the greatest height the particle reaches ? **2**

(d) Show that $\dfrac{x}{e}>\ln x$ for $x>e$. **2**

Question 15 (15 marks) **Marks**

(a) Solve the equation $z^2 = |z|^2 - 4$ **3**

(b) Use mathematical induction to show that $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ for all positive integers $n \geq 1$. **4**

(c) Position vectors of the points A, B and C, relative to an origin O, are $-\underset{\sim}{i} - \underset{\sim}{j}$, $\underset{\sim}{j} + 2\underset{\sim}{k}$, and $4\underset{\sim}{i} + \underset{\sim}{k}$ respectively.

(i) Find $\overrightarrow{AB}$. **1**

(ii) Find $|\overrightarrow{AB}|$. **1**

(iii) Prove that $\angle ABC$ is a right angle. **3**

(d) A particle of mass m is moving in a straight line under the action of a force. **3**

$$F = \frac{m}{x^3}(6 - 10x)$$

What is the velocity in any position, if the particle starts from rest at $x = 1$?

Question 16 (15 marks) **Marks**

(a) (i) Let $I_n = \int_0^1 (1 - x^r)^n dx$ where $r > 0$ for $n = 1, 2, 3, \ldots$ **2**

Show that $I_n = \frac{nr}{nr + 1} I_{n-1}$

(ii) Hence or otherwise, find the exact value of $\int_0^1 (1 - x^{\frac{3}{2}})^3 dx$ **2**

(b) Let a, b and c be real numbers such that $a > b > c > 1$.

(i) Show that $a^{a-b} b^{b-c} > c^{a-c}$. **1**

(ii) Hence show that $a^a b^b c^c > a^b b^c c^a$. **1**

(c) Sketch the locus of z on the Argand diagram where the inequalities $|z - 1| \leq 3$ and $\text{Im}(z) \geq 3$ hold simultaneously. **3**

(d) The point A, with coordinates $(0, a, b)$ lies on the line l_1, which has the equation:

l_1: $\underset{\sim}{r} = 6\underset{\sim}{i} + 19\underset{\sim}{j} - \underset{\sim}{k} + \lambda(\underset{\sim}{i} + 4\underset{\sim}{j} - 2\underset{\sim}{k})$

(i) Find the values of a and b. **2**

(ii) The point P lies on l_1 and is such that OP is perpendicular to l_1, where O is the origin. Find the position vector of point P. **4**

End of paper

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$l = \frac{\theta}{360} \times 2\pi r$

Area

$A = \frac{\theta}{360} \times \pi r^2$

$A = \frac{h}{2}(a + b)$

Surface area

$A = 2\pi r^2 + 2\pi rh$

$A = 4\pi r^2$

Volume

$V = \frac{1}{3}Ah$

$V = \frac{4}{3}\pi r^3$

Functions

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

For $ax^3 + bx^2 + cx + d = 0$:

$\alpha + \beta + \gamma = -\frac{b}{a}$

$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$(x - h)^2 + (y - k)^2 = r^2$

Financial Mathematics

$A = P(1 + r)^n$

Sequences and series

$T_n = a + (n - 1)d$

$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$

$T_n = ar^{n-1}$

$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1},\ r \neq 1$

$S = \frac{a}{1 - r},\ |r| < 1$

Logarithmic and Exponential Functions

$\log_a a^x = x = a^{\log_a x}$

$\log_a x = \frac{\log_b x}{\log_b a}$

$a^x = e^{x \ln a}$

Trigonometric Functions

$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$

$A = \frac{1}{2}ab\sin C$

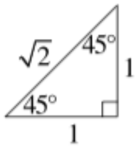
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

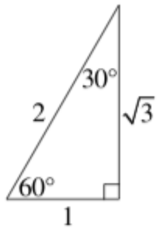
$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$l = r\theta$

$A = \frac{1}{2}r^2\theta$





Trigonometric identities

$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$

$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$

$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$

$\cos^2 x + \sin^2 x = 1$

Compound angles

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

$\cos(A + B) = \cos A\cos B - \sin A\sin B$

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1 + t^2}$

$\cos A = \frac{1 - t^2}{1 + t^2}$

$\tan A = \frac{2t}{1 - t^2}$

$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$

$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$

$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$

$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$

$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$

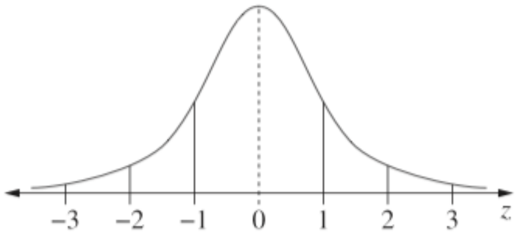
$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$

Statistical Analysis

$z = \frac{x - \mu}{\sigma}$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$E(X) = \mu$

$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$

Probability

$P(A \cap B) = P(A)P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$

Continuous random variables

$P(X \leq x) = \int_a^x f(x)\,dx$

$P(a < X < b) = \int_a^b f(x)\,dx$

Binomial distribution

$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$

$X \sim \text{Bin}(n, p)$

$\Rightarrow \ P(X = x)$

$= \binom{n}{x}p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$

$E(X) = np$

$\text{Var}(X) = np(1 - p)$

Differential Calculus

Function	**Derivative**
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-\left[f(x)\right]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+\left[f(x)\right]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2+y^2}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

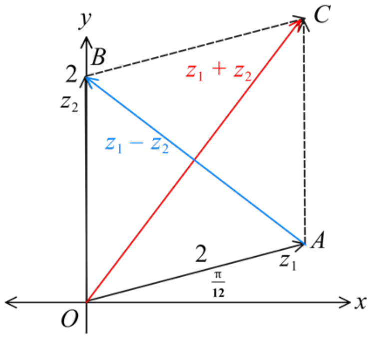
$$\ddot{x} = -n^2(x-c)$$

ACE Examination 2023
Year 12 Mathematics Extension 2 Yearly Examination
Worked solutions and marking guidelines

Section I		
	Solution	**Criteria**
1	Let $u = x^2 + 1$ $\frac{du}{dx} = 2x$ and $xdx = \frac{1}{2}du$ $\int \frac{x}{\sqrt[3]{x^2+1}}dx = \frac{1}{2}\int \frac{1}{\sqrt[3]{u}}du$ $= \frac{1}{2}\int u^{-\frac{1}{3}}du$ $= \frac{1}{2} \times \frac{3}{2}u^{\frac{2}{3}} + C$ $= \frac{3}{4}\sqrt[3]{(x^2+1)^2} + C$	1 Mark: C
2	$\big(z-(2+3i)\big)\big(z-(2-3i)\big) = 0$ $z^2 - 2z + 3iz - 2z - 3iz + 4 + 9 = 0$ $\therefore z^2 - 4z + 13 = 0$	1 Mark: A
3	$\underset{\sim}{u} \, . \, \underset{\sim}{v} = 1 \times 4 + (-1) \times 12 + (-1) \times (-3)$ $= -5$	1 Mark: A
4	$z = 3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ $\bar{z} = 3\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$ $\bar{z}^3 = 3^3\left(\cos\frac{3\pi}{6} - i\sin\frac{3\pi}{6}\right)$ $= 27\left(\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}\right)$	1 Mark: C
5	$\sqrt{x^2+y^2+z^2} = \sqrt{1^2+(-3)^2+5^2}$ $= \sqrt{35}$	1 Mark: B

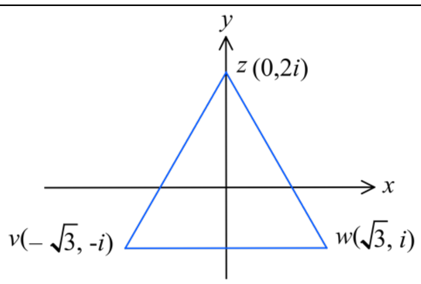
6	$v^2 + 9x^2 = k$ $v^2 = k - 9x^2$ $\ddot{x} = \frac{d}{dx}\left(\frac{v^2}{2}\right)$ $= \frac{d}{dx}\left(\frac{k}{2} - \frac{9x^2}{2}\right)$ $= -9x = -n^2x$ Hence $n = 3$ *Period* $T = \frac{2\pi}{n} = \frac{2\pi}{3}$	1 Mark: D
7	The converse of a statement 'If P then Q' is 'If Q then P'. The statements can be represented as: the converse of $P \Rightarrow Q$ is $Q \Rightarrow P$ or $P \Leftarrow Q$. The converse of a true statement need not be true.	1 Mark: A
8	$v = f(x)$ $a = v\frac{dv}{dx} = f(x)f'(x)$	1 Mark: D
9	Using the conjugate root theorem $(2 + i)$ and $(2 - i)$ are both roots of the equation $z^3 + pz + q = 0$. $(2 + i) + (2 - i) + \alpha = 0$ (sum of the roots) $\alpha = -4$ $(2 + i) \times (2 - i) \times (-4) = -q$ (product of the roots) $(4 + 1) \times (-4) = -q$ $q = 20$ $(2 + i)(2 - i) + (2 - i) \times (-4) + (2 + i) \times (-4) = p$ $p = -11$ $\therefore p = -11$ and $q = 20$	1 Mark: B
10	Use the substitution $u = \cos x$ $\frac{du}{dx} = -\sin x$ $du = -\sin x dx$ When $x = 0$ then $u = 1$ and when $x = \frac{\pi}{2}$ then $u = 0$ $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sqrt{1 - \cos^2 x}} dx = \int_1^0 \frac{-1}{\sqrt{1 - u^2}} du$ $= [\sin^{-1} u]_0^1$ $= \frac{\pi}{2}$	1 Mark: D

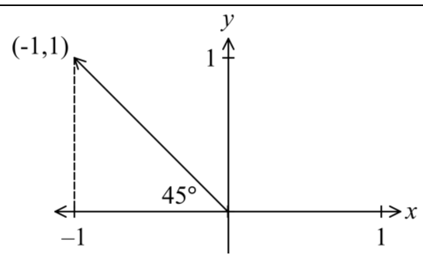
Section II

	Solution	**Criteria**
11(a) (i)	$\overline{iz} = \overline{i(4-i)} = \overline{4i+1}$ $= 1-4i$	1 Mark: Correct answer.
11(a) (ii)	$\frac{1}{z} = \frac{1}{4-i} \times \frac{4+i}{4+i} = \frac{4+i}{16+1}$ $= \frac{4}{17} + \frac{1}{17}i$	1 Mark: Correct answer.
11(b)	$\left\|\overrightarrow{OA}\right\| = \sqrt{2^2+6^2+(-3)^2}$ $= \sqrt{49} = 7$ $\widehat{OA} = \frac{\overrightarrow{OA}}{\left\|\overrightarrow{OA}\right\|}$ $= \frac{1}{7}(2\underset{\sim}{i} + 6\underset{\sim}{j} - 3\underset{\sim}{k})$	2 Marks: Correct answer. 1 Mark: Finds the magnitude of $\overrightarrow{OA}$.
11(c)	Let $u = x^2+4$ $\frac{du}{dx} = 2x \;\; or \;\; \frac{1}{2}du = xdx$ When $x = 0$ then $u = 4$ and when $x = \sqrt{5}$ then $u = 9$ $\int_0^{\sqrt{5}} \frac{x}{\sqrt{x^2+4}}dx = \frac{1}{2}\int_4^9 \frac{1}{\sqrt{u}}du = \frac{1}{2}\int_4^9 u^{-\frac{1}{2}}du$ $= \frac{1}{2}\left[2u^{\frac{1}{2}}\right]_4^9$ $= [\sqrt{9}-\sqrt{4}]$ $= 1$	2 marks: Correct answer. 1 mark: Sets up the integral in terms of u.
11(d) (i)		1 Mark: Correct answer.
11(d) (ii)	See Argand diagram above.	1 Mark: Correct answer.
11(d) (iii)	Vectors OC and AB form a parallelogram However $OA = OB = 2$. Hence $OBCA$ is a rhombus. Diagonal of a rhombus bisects the angle through which it passes $\therefore \angle AOC = \frac{1}{2} \times \left(\frac{\pi}{2} - \frac{\pi}{12}\right) = \frac{5\pi}{24}$ $\arg(z_1+z_2) = \frac{\pi}{12} + \frac{5\pi}{24}$ $= \frac{7\pi}{24}$	2 Marks: Correct answer. 1 Mark: Shows some understanding.

11(e)	$\int \frac{1}{\sqrt{12+4x-x^2}}dx = \int \frac{1}{\sqrt{12-(x^2-4x)}}dx$ $= \int \frac{1}{\sqrt{16-(x^2-4x+4)}}dx$ $= \int \frac{1}{\sqrt{16-(x-2)^2}}dx$ $= \sin^{-1}\left(\frac{x-2}{4}\right) + C$	2 Marks: Correct answer. 1 Mark: Completes the square.
11(f)	Step 1: To prove true for $n = 1$ $\text{LHS} = 1 + x$ $\text{RHS} = \frac{x^{1+1}-1}{x-1} = \frac{(x+1)(x-1)}{(x-1)} = 1 + x$ Result is true for $n = 1$ Step 2: Assume true for $n = k$ $S_k = \frac{x^{k+1}-1}{x-1}$ Step 3: To prove true for $n = k + 1$ $S_{k+1} = \frac{x^{k+2}-1}{x-1}$ $S_k + T_{k+1} = S_{k+1}$ $\text{LHS} = \frac{x^{k+1}-1}{x-1} + x^{k+1}$ $= \frac{x^{k+1}-1}{x-1} + \frac{x^{k+1}(x-1)}{(x-1)}$ $= \frac{x^{k+1}-1+x^{k+2}-x^{k+1}}{x-1}$ $= \frac{x^{k+2}-1}{x-1}$ $= \text{RHS}$ Step 4: True by induction	3 marks: Correct answer. 2 marks: Proves the result true for $n = 1$ and attempts to use the result of $n = k$ to prove the result for $n = k + 1$ 1 mark: Proves the result true for $n = 1$.
12(a)	$\int x\ e^{-x}dx$ $u = x \quad \frac{du}{dx} = 1$ $\frac{dv}{dx} = e^{-x} \quad v = -e^{-x}$ $\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$ $\int x\ e^{-x}dx = x \times (-e^{-x}) - \int -e^{-x} \times 1\, dx$ $= -xe^{-x} + \int e^{-x}\, dx$ $= -xe^{-x} - e^{-x} + C$	3 Marks: Correct answer. 2 Marks: Makes significant progress. 1 Mark: Correctly applies integration by parts.
12(b)(i)	$(1+ia)^4 = 1 + 4ia - 6a^2 - 4ia^3 + a^4$	1 Mark: Correct answer.

12(b) (ii)	$(1+ia)^4$ is real if $4a-4a^3=0$ Then $4a(1-a^2)=0$ $\therefore a=0,\pm 1$	2 Marks: Correct answer. 1 Mark: Finds an equation for a or equivalent merit.
12(c) (i)	$\dfrac{5x^3-3x^2+2x-1}{x^2(x^2+1)}=\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{Cx+D}{x^2+1}$ Using partial fractions to find A, B, C and D $5x^3-3x^2+2x-1=Ax(x^2+1)+B(x^2+1)+(Cx+D)x^2$ $=Ax^3+Ax+Bx^2+B+Cx^3+Dx^2$ $=(A+C)x^3+(B+D)x^2+Ax+B$ Equating the coefficients $A+C=5$ ① $B+D=-3$ ② $A=2$ and $B=-1$ Equation ① $2+C=5$ or $C=3$ Equation ② $B+D=-3$ or $D=-2$ $\therefore A=2, B=-1, C=3$ and $D=-2$	2 Marks: Correct answer. 1 Mark: Makes progress in finding A, B, C or D.
12(c) (ii)	$\displaystyle\int \frac{5x^3-3x^2+2x-1}{x^2(x^2+1)}dx=\int \frac{2}{x}-\frac{1}{x^2}+\frac{3x-2}{x^2+1}dx$ $\displaystyle=\int \frac{2}{x}-\frac{1}{x^2}+\frac{3x}{x^2+1}-\frac{2}{x^2+1}dx$ $\displaystyle=2\ln\|x\|+\frac{1}{x}+\frac{3}{2}\ln(x^2+1)-2\tan^{-1}x+C$	2 Marks: Correct answer. 1 Mark: Correctly finds one of the integrals.
12(d) (i)	$\ddot{x}=-12\sin 2t$ $\dot{x}=-12\displaystyle\int \sin 2t\,dt$ $=-12\displaystyle\int 2\sin t\cos t\,dt$ $=-12\sin^2 t+C$ *When* $t=0$, $\dot{x}=6$ hence $C=6$ $\therefore \dot{x}=-12\sin^2 t+6$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
12(d) (ii)	$x=\displaystyle\int -12\sin^2 t+6\,dt$ $=\displaystyle\int -12\left(\frac{1-\cos 2t}{2}\right)+6\,dt=\int -6+6\cos 2t+6\,dt$ $=\displaystyle\int 6\cos 2t\,dt$ $=3\sin 2t+C$ *When* $t=0, x=0$ hence $C=0$ $\therefore x=3\sin 2t$ Then $\ddot{x}=-12\sin 2t$ $=-4\times 3\sin 2t$ $=-4x$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Finds $x=3\sin 2t+C$ or equivalent merit.

13(a)	$\int \frac{1}{9x^2+6x+5}dx = \int \frac{dx}{9\left[\left(x^2+\frac{2}{3}x\right)\right]+5}$ $= \int \frac{dx}{9\left[\left(x+\frac{1}{3}\right)^2-\frac{1}{9}\right]+5}$ $= \int \frac{dx}{9\left(x+\frac{1}{3}\right)^2+4}$ $= \int \frac{dx}{(3x+1)^2+2^2}$ $= \frac{1}{3}\int \frac{3dx}{(3x+1)^2+2^2}$ $= \frac{1}{6}\tan^{-1}\frac{(3x+1)}{2}+C$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Shows some understanding.
13(b)(i)	Line l_1 intersects the line l_2 then: $(11\underset{\sim}{i}+2\underset{\sim}{j}+17\underset{\sim}{k})+\lambda(-2\underset{\sim}{i}+\underset{\sim}{j}-4\underset{\sim}{k})$ $= (-5\underset{\sim}{i}+11\underset{\sim}{j}+p\underset{\sim}{k})+\mu(-3\underset{\sim}{i}+2\underset{\sim}{j}+2\underset{\sim}{k})$ $11-2\lambda=-5-3\mu$ ① $2+\lambda=11+2\mu$ ② $17-4\lambda=p+2\mu$ ③ Equation ① + 2 × ② $15=17+\mu$ then $\mu=-2$ Equation ② $2+\lambda=11-4$ then $\lambda=5$ Equation ③ $17-4\times 5=p+2\times -2$ $p=17-20+4$ $=1$ $\therefore$ The value of p is 1.	3 Marks: Correct answer. 2 Marks: Finds the correct values for λ and μ. 1 Mark: Shows some understanding.
13(b)(ii)	From 13(b)(i) $\lambda=5$, $\mu=-2$ and $p=1$ l_1: $(11\underset{\sim}{i}+2\underset{\sim}{j}+17\underset{\sim}{k})+5(-2\underset{\sim}{i}+\underset{\sim}{j}-4\underset{\sim}{k})=\underset{\sim}{i}+7\underset{\sim}{j}+-3\underset{\sim}{k}$ or l_2: $(-5\underset{\sim}{i}+11\underset{\sim}{j}+\underset{\sim}{k})+-2(-3\underset{\sim}{i}+2\underset{\sim}{j}+2\underset{\sim}{k})=\underset{\sim}{i}+7\underset{\sim}{j}+-3\underset{\sim}{k}$ $\therefore$ Point of intersection is $\underset{\sim}{i}+7\underset{\sim}{j}+-3\underset{\sim}{k}$.	1 Mark: Correct answer.
13(c)	Pythagoras theorem $zw=\sqrt{3^2+(\sqrt{3})^2}$ $=\sqrt{12}=2\sqrt{3}$ $zv=\sqrt{3^2+(\sqrt{3})^2}$ $=\sqrt{12}=2\sqrt{3}$ $wv=\sqrt{3}+\sqrt{3}$ $=2\sqrt{3}$ $\therefore$ Equilateral triangle.  	2 Marks: Correct answer. 1 Mark: Plots the points or makes some progress.

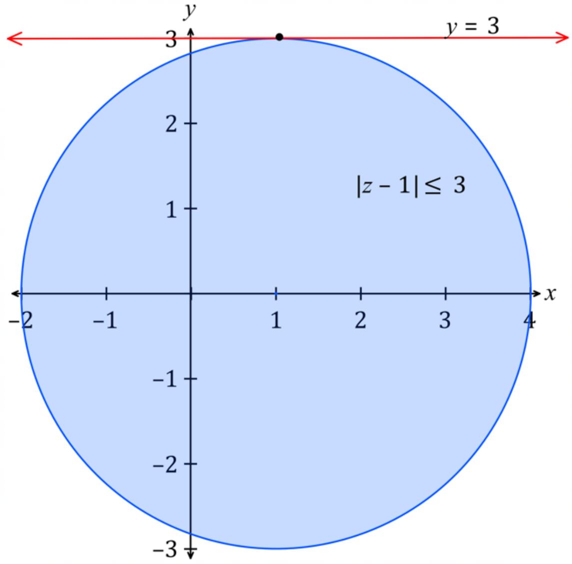
13(d)(i)	Amplitude of motion is 2 cm (a = 2) Period of motion is 4 seconds $T = \frac{2\pi}{n} = 4$ $n = \frac{\pi}{2}$ $v^2 = n^2(a^2 - x^2)$ $= \left(\frac{\pi}{2}\right)^2 (2^2 - x^2)$ Maximum velocity occurs when x = 0 $v^2 = \left(\frac{\pi}{2}\right)^2 \times 2^2$ $v = \pi$ cms^{-1}	2 Marks: Correct answer. 1 Mark: Finds the values of a and n or shows some understanding of the problem.
13(d)(ii)	Displacement is $x = \pm 1$ cm $v^2 = \left(\frac{\pi}{2}\right)^2 (2^2 - (\pm 1)^2)$ $= \frac{3\pi^2}{4}$ $v = \pm \frac{\sqrt{3}}{2}\pi$ cms^{-1}	2 Marks: Correct answer. 1 Mark: Makes some progress towards the solution.
13(e)	Equations of projectile motion $x = Vt\cos\alpha$ $5 = 10t\cos\alpha$ $t = \frac{1}{2\cos\alpha}$ ① $y = -\frac{1}{2}gt^2 + Vt\sin\alpha$ $6.25 = -\frac{1}{2} \times 10 \times t^2 + 10t\sin\alpha$ $25 = -20t^2 + 40t\sin\alpha$ ② Substituting equation ① into equation ② $25 = -20 \times \left(\frac{1}{2\cos\alpha}\right)^2 + 40 \times \left(\frac{1}{2\cos\alpha}\right) \times \sin\alpha$ $25 = -5\sec^2\alpha + 20\tan\alpha$ $5 = -(\tan^2\alpha + 1) + 4\tan\alpha$ $\tan^2\alpha - 4\tan\alpha + 4 = 0$ $(\tan\alpha - 2)^2 = 0$ $\tan\alpha = 2$ $\alpha = \tan^{-1}2$ $= 63°26'$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
14(a)(i)	$r^2 = 1^2 + 1^2$ $r = \sqrt{2}$ $\tan\theta = \frac{1}{-1}$ $\theta = \frac{3\pi}{4}$  $\therefore -1 + i = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$	2 Marks: Correct answer. 1 Mark: Correctly determines the argument or the modulus.

14(a) (ii)	$(-1+i)^n = (\sqrt{2})^n\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)^n$ $= 2^{\frac{n}{2}}\left(\cos\frac{3n\pi}{4} + i\sin\frac{3n\pi}{4}\right)$	1 Mark: Correct answer.
14(b)	$\frac{1}{a}+\frac{1}{b}-\frac{4}{m} = \frac{1}{a}+\frac{1}{b}-\frac{4}{a+b}$ $= \frac{b(a+b)+a(a+b)-4ab}{ab(a+b)}$ $= \frac{(a+b)^2-4ab}{ab(a+b)}$ $= \frac{a^2-2ab+b^2}{ab(a+b)}$ $= \frac{(a-b)^2}{ab(a+b)}$ Given $a>0, b>0$ then $ab(a+b)>0$ Also $(a-b)^2>0$ for all values of a and b Hence $\frac{1}{a}+\frac{1}{b}-\frac{4}{m} = \frac{(a-b)^2}{ab(a+b)} > 0$ $\therefore \frac{1}{a}+\frac{1}{b} > \frac{4}{m}$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Shows some understanding.
14(c) (i)	Cartesian equation of the path $y = x\tan\theta - \frac{gx^2}{2v^2}\sec^2\theta$ Passes through (b,h) and (c,h) $h = b\tan\theta - \frac{gb^2}{2v^2}\sec^2\theta$ ① $h = c\tan\theta - \frac{gc^2}{2v^2}\sec^2\theta$ ② Equating equations ① and ② $b\tan\theta - \frac{gb^2}{2v^2}\sec^2\theta = h = c\tan\theta - \frac{gc^2}{2v^2}\sec^2\theta$ $(b-c)\tan\theta = (b^2-c^2)\frac{g}{2v^2}\sec^2\theta$ $\tan\theta = (b+c)\frac{g}{2v^2}\sec^2\theta$ $\therefore v^2 = \frac{(b+c)g\sec^2\theta}{2\tan\theta}$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Uses the cartesian equation of the path with either (b,h) or (c,h).
14(c) (ii)	Substitute the result for v^2 into equation ① $h = b\tan\theta - \frac{gb^2\sec^2\theta}{2} \times \frac{2\tan\theta}{(b+c)g\sec^2\theta}$ $h = b\tan\theta - \frac{b^2\tan\theta}{b+c}$ $h(b+c) = (b^2+bc)\tan\theta - b^2\tan\theta$ $= bc\tan\theta$ $\therefore \tan\theta = \frac{h(b+c)}{bc}$	2 Marks: Correct answer. 1 Mark: Shows some understanding.

14(c) (iii)	Greatest height at $x = \frac{b+c}{2}$ $y = \left(\frac{b+c}{2}\right)\tan\theta - \left(\frac{b+c}{2}\right)^2 \times \frac{g\sec^2\theta\ 2\tan\theta}{2(b+c)g\sec^2\theta}$ $= \left(\frac{b+c}{2}\right)\tan\theta - \left(\frac{b+c}{4}\right) \times \tan\theta$ $= \left(\frac{b+c}{4}\right)\tan\theta$ $= \left(\frac{b+c}{4}\right)\frac{h(b+c)}{bc}$ $y = \frac{h(b+c)^2}{4bc}$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
14(d)	Let $f(x) = \frac{x}{e} - \ln x$ for $x > e$ Hence $f(e) = 0$ and $f(x)$ is an increasing function $x > e$ $\therefore f(x) > 0$ for $x > e$ $\therefore \frac{x}{e} > \ln x$ for $x > e$	2 Marks: Correct answer. 1 Mark: Uses $f(e) = 0$ to deduce required inequality.
15(a)	Let $z = x + iy$ Then $z^2 = (x+iy)^2 = x^2 + 2ixy - y^2$ $z^2 = \lvert z\rvert^2 - 4$ $x^2 + 2ixy - y^2 = \lvert x+iy\rvert^2 - 4$ $x^2 + 2ixy - y^2 = \left(\sqrt{x^2+y^2}\right)^2 - 4$ $x^2 + 2ixy - y^2 = x^2 + y^2 - 4$ $2ixy - y^2 = y^2 - 4$ $2y^2 - 2ixy - 4 = 0$ $2(y^2 - ixy - 2) = 0$ $(y^2 - 2) = 0$ then $y = \pm\sqrt{2}$ $ixy = 0$ then $x = 0$ Therefore $z = x + iy$ $z = \pm\sqrt{2}i$	3 Marks: Correct answer. 2 Marks: Makes significant progress towards the solution. 1 Mark: Writes the equation using $z = x + iy$ or equivalent merit.

15(b)	Step 1: To prove true for $n = 1$ $\text{LHS} = (\cos\theta + i\sin\theta)^1 = \cos\theta + i\sin\theta$ $\text{RHS} = \cos 1\theta + i\sin 1\theta = \cos\theta + i\sin\theta$ Result is true for $n = 1$ Step 2: Assume true for $n = k$ $(\cos\theta + i\sin\theta)^k = \cos k\theta + i\sin k\theta$ Step 3: To prove true for $n = k + 1$ $(\cos\theta + i\sin\theta)^{k+1} = \cos(k+1)\theta + i\sin(k+1)\theta$ $\text{LHS} = (\cos\theta + i\sin\theta)^{k+1}$ $= (\cos\theta + i\sin\theta)^k \times (\cos\theta + i\sin\theta)$ $= (\cos k\theta + i\sin k\theta)(\cos\theta + i\sin\theta)$ $= (\cos k\theta\cos\theta - \sin k\theta\sin\theta) + i(\sin k\theta\cos\theta + \cos k\theta\sin\theta)$ $= \cos(k\theta + \theta) + i\sin(k\theta + \theta)$ $= \cos(k+1)\theta + i\sin(k+1)\theta$ $= \text{RHS}$ Step 4: True by induction	4 Marks: Correct answer. 3 Marks: Makes significant progress towards the solution. 2 Marks: Proves the result true for $n = 1$ and attempts to use the result of $n = k$ to prove the result for $n = k + 1$ 1 Mark: Proves the result true for $n = 1$
15(c) (i)	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ $= (\underset{\sim}{j} + 2\underset{\sim}{k}) - (-\underset{\sim}{i} - \underset{\sim}{j})$ $= \underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$	1 Mark: Correct answer.
15(c) (ii)	$\lvert\overrightarrow{AB}\rvert = \sqrt{x^2 + y^2 + z^2}$ $= \sqrt{1^2 + 2^2 + 2^2}$ $= 3$	1 Mark: Correct answer.
15(c) (iii)	$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$ $= (4\underset{\sim}{i} + \underset{\sim}{k}) - (\underset{\sim}{j} + 2\underset{\sim}{k})$ $= 4\underset{\sim}{i} - \underset{\sim}{j} - \underset{\sim}{k}$ $\lvert\overrightarrow{BC}\rvert = \sqrt{x^2 + y^2 + z^2}$ $= \sqrt{4^2 + (-1)^2 + (-1)^2}$ $= 3\sqrt{2}$ $\cos\angle CAB = \dfrac{\overrightarrow{AB} \cdot \overrightarrow{BC}}{\lvert\overrightarrow{AB}\rvert\lvert\overrightarrow{BC}\rvert}$ $= \dfrac{1 \times 4 + 2 \times (-1) + 2 \times (-1)}{3 \times 3\sqrt{2}}$ $= \dfrac{0}{9\sqrt{2}} = 0$ $\angle CAB = 90°$	3 Marks: Correct answer. 2 Marks: Uses the angle between two vectors. 1 Mark: Finds $\overrightarrow{BC}$ or $\lvert\overrightarrow{BC}\rvert$.

15(d)	$F=\frac{m}{x^3}(6-10x)$ $ma=\frac{m}{x^3}(6-10x)$ $v\frac{dv}{dx}=\frac{6}{x^3}-\frac{10}{x^2}$ $\int vdv=\int\left(\frac{6}{x^3}-\frac{10}{x^2}\right)dx$ $\frac{1}{2}v^2=\left(\frac{6x^{-2}}{-2}-\frac{10x^{-1}}{-1}\right)+C$ $\frac{1}{2}v^2=\left(\frac{-3}{x^2}+\frac{10}{x}\right)+C$ When $v=0$ and $x=1$ $\frac{1}{2}0^2=\left(\frac{-3}{1^2}+\frac{10}{1}\right)+C$ $C=-7$ Hence $\frac{1}{2}v^2=\left(\frac{-3}{x^2}+\frac{10}{x}\right)-7$ $v^2=\left(\frac{-6}{x^2}+\frac{20}{x}\right)-14$ $=\frac{-6+20x-14x^2}{x^2}$ $v=\pm\frac{1}{x}\sqrt{2(-3+10x-7x^2)}$	3 Marks: Correct answer. 2 Marks: Integrates and uses the initial conditions to find an expression for $\frac{1}{2}v^2$. 1 Mark: Uses $v\frac{dv}{dx}$ for acceleration.
16(a) (i)	$I_n=\int_0^1(1-x^r)^n dx$ $=[x(1-x^r)^n]_0^1-n\int_0^1 x(1-x^r)^{n-1}(-rx^{r-1})dx$ $=0-nr\int_0^1[(1-x^r)^n-1](1-x^r)^{n-1}dx$ $=0-nr\int_0^1(1-x^r)^n-(1-x^r)^{n-1}dx$ $I_n=nr(-I_n+I_{n-1})$ $(nr+1)I_n=nrI_{n-1}$ $I_n=\frac{nr}{nr+1}I_{n-1}$	2 Marks: Correct answer. 1 Mark: Sets up the integration by parts.
16(a) (ii)	For $r=\frac{3}{2}$ and $n=3$ $I_3=\frac{3\times\frac{3}{2}}{3\times\frac{3}{2}+1}\times I_2$ $I_2=\frac{2\times\frac{3}{2}}{2\times\frac{3}{2}+1}\times I_1$ $I_1=\frac{1\times\frac{3}{2}}{1\times\frac{3}{2}+1}\times I_0$ But $I_0=\int_0^1(1-x^r)^0dx=\int_0^1 1dx=1$ $I_3=\frac{3\times\frac{3}{2}}{3\times\frac{3}{2}+1}\times\frac{2\times\frac{3}{2}}{2\times\frac{3}{2}+1}\times\frac{1\times\frac{3}{2}}{1\times\frac{3}{2}+1}\times 1=\frac{81}{220}$	2 Marks: Correct answer. 1 Mark: Shows some understanding.
16(b) (i)	$a^{a-b}b^{b-c}>c^{a-b}c^{b-c}=c^{a-c}$ since $a>b>c>1$ $\therefore a^{a-b}b^{b-c}>c^{a-c}$	1 Mark: Correct answer.
16(b) (ii)	$(a^b b^c c^c)\times a^{a-b}b^{b-c}>(a^b b^c c^c)\times c^{a-c}$ $\therefore a^a b^b c^c>a^b b^c c^a$	1 Mark: Correct answer.

16(c)	$\lvert z-1\rvert \leq 3$ represents a region with centre (1, 0) and radius less than or equal to 3. $\text{Im}(z) \geq 3$ represents a region above the horizontal line $y = 3$. The point (1,3) is where the two inequalities hold. 	3 Marks: Correct answer. 2 Marks: Correctly graphs one inequality. 1 Mark: Makes some progress.
16(d)(i)	$\underset{\sim}{i}$ component $6+\lambda=0$ $\lambda=-6$ $\underset{\sim}{j}$ component $19-6\times 4=a$ $a=-5$ $\underset{\sim}{k}$ component $-1-6\times(-2)=b$ $\therefore\ a=-5$ and $b=11$.	2 Marks: Correct answer. 1 Mark: Finds λ or shows some understanding.
16(d)(ii)	$\overrightarrow{OP}=(6+\lambda)\underset{\sim}{i}+(19+4\lambda)\underset{\sim}{j}+(-1-2\lambda)\underset{\sim}{k}$ Direction vector of l_1: $\underset{\sim}{i}+4\underset{\sim}{j}-2\underset{\sim}{k}$ $\overrightarrow{OP}$ and l_1 are perpendicular $\left[(6+\lambda)\underset{\sim}{i}+(19+4\lambda)\underset{\sim}{j}+(-1-2\lambda)\underset{\sim}{k}\right]\cdot(\underset{\sim}{i}+4\underset{\sim}{j}-2\underset{\sim}{k})=0$ Hence $6+\lambda+(19+4\lambda)4+(-1-2\lambda)-2=0$ $6+\lambda+76+16\lambda+2+4\lambda=0$ $21\lambda+84=0$ $\lambda=-4$ Therefore $\overrightarrow{OP}=(6-4)\underset{\sim}{i}+(19+4\times(-4))\underset{\sim}{j}+(-1-2\times(-4))\underset{\sim}{k}$ $=2\underset{\sim}{i}+3\underset{\sim}{j}+7\underset{\sim}{k}$	4 Marks: Correct answer. 3 Marks: Makes significant progress towards the solution. 2 Marks: Applies the statement for perpendicular vectors. 1 Mark: Shows some understanding.